Teacher notes
Topic A

Circular motion and work done

It is a common misconception that the definition of work is force times displacement. Applied to any motion that starts and ends at the same point in space (displacement = zero) this would imply that zero work is being done. So pushing a heavy piece of furniture around and returning it to its starting point should not get you tired at all!

Applied to circular motion we have a similar situation: in a full revolution the displacement is zero and so the work done by *any* force acting on the body should be zero according to the incorrect definition of work.

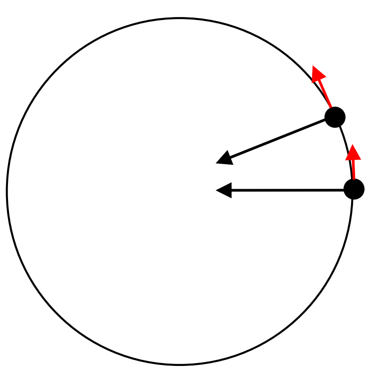
The definition of work says that for a force F that is *constant in magnitude and direction* and for a body that is moved along a *straight line* by a distance s the work done is

$$W = Fs\cos\theta$$

where θ is the angle between the direction of F and the straight line along which the body moves.

But if the motion does not take place along a straight line or when the force is not constant we have to be careful.

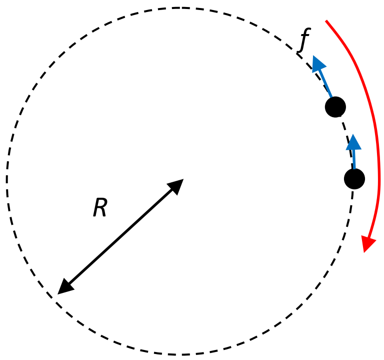
In circular motion, say a body attached to a string that makes the body rotate along a horizontal circle we have the tension of the string as the net force on the body. Here the force keeps changing direction but is constant in magnitude.



The motion along the circle can be thought of as a sequence of very small straight line segments. Along any one of these segments the force is at right angles to the motion. Because the segment is very small, we may take the force to be constant in both magnitude and direction along the segment and so

$W = Fs\cos\theta$ applies. But now $\theta = 90^\circ$ and so $W = 0$ along this segment. It is zero along any other segment and so $W = 0$ for motion along any part of the circle, not just a full revolution.

We can also think of circular motion with more forces in addition to that pointing towards the centre. For example a constant (in magnitude) frictional force *f* that opposes the motion.



Splitting the circle into straight line segments as before we see that along each segment the work done by the frictional force is $W = f\Delta s\cos 180^\circ = -f\Delta s$ where Δs is the length of the tiny straight segment. Adding the work along all the segments we find that the total work done is

$$W = -\sum f\Delta s = -f\sum \Delta s = -fL$$

where *L* is the total distance travelled (distance, not displacement). For a full revolution, $L = 2\pi R$ and $W = -2\pi fR$.

Class example

A body of mass 2.0 kg moves along a circular horizontal path of radius 0.63 m with speed 6.0 m s^{-1}. A small frictional force of 3.0 N opposite to the velocity will bring the body to rest. How many revolutions will the body to before stopping?

The work done by the frictional force is the change in kinetic energy and so equals

$$0 - \frac{1}{2}mv^2 = -\frac{1}{2}\times 2.0 \times 6.0^2 = -36 \text{ J}$$

Thus, $-fL = -36 \text{ J} \Rightarrow 3.0L = 36 \Rightarrow L = 12 \text{ m}$. This corresponds to $\dfrac{12}{2\pi R} = \dfrac{12}{2\pi \times 0.63} = 3.0 \text{ rev}$.